Exercise 12

For the following exercises, find $f^{-1}(x)$ for each function.

$$f(x) = \frac{2x+3}{5x+4}$$

Solution

Switch the roles of x and y.

$$x = \frac{2y+3}{5y+4}$$

Solve for y.

$$x \times (5y+4) = \frac{2y+3}{5y+4} \times (5y+4)$$

$$5xy + 4x = 2y + 3$$

$$5xy + 4x - 2y = 3$$

$$5xy - 2y = 3 - 4x$$

$$(5x-2)y = 3 - 4x$$

$$y = \frac{3-4x}{5x-2}$$

Therefore,

$$f^{-1}(x) = \frac{3-4x}{5x-2}.$$